this time: measurement error; prob. models for means

read: LN pp. 137–173 ①

AMS 7
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next time: interval estimation

today: LN pp. 127 →

ex. TJ "1 lb package of butter"

$$\begin{bmatrix} 16 \text{ oz} \\ 16 \\ 16 \\ \vdots \\ 16 \end{bmatrix} \quad \begin{bmatrix} 16.0 \\ 16.0 \\ \vdots \\ 16.0 \end{bmatrix} \quad \begin{bmatrix} 15.98 \\ 16.01 \\ 15.97 \\ \vdots \\ 16.00 \end{bmatrix}$$

deterministic: you always get the same thing

observable

not observable

$$Y_1 = \theta + b + e_1$$

$$Y_2 = \theta + b + e_2$$

$$\vdots$$

$$Y_n = \theta + b + e_n$$

probabilistic (stochastic): the measurement varies haphazardly from repetition to repetition

"at random"

Basic measurement error model ②

Y_i		θ	+	b	+	e_i
(observation 1)	=	(true value)	+	(bias)	+	("random" error 1)
(15.98)		16.00		0.00		−0.02
(observation 2)	=	(true value)	+	(bias)	+	("random" error 2)
(16.01)		16.00		0.00		+0.01
⋮		⋮		⋮		⋮
(observation n)	=	(true value)	+	(bias)	+	("random" error n)
(16.00)		16.00		0.00		+.00

assumptions:
errors have mean 0
errors are IID

$$Y_1 = \theta + b + e_1$$
$$Y_2 = \theta + b + e_2$$
$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$
$$Y_n = \theta + b + e_n$$

$$\frac{(-0.02) + (+.01) + \dots + (+0.00)}{n}$$

take mean

$$\bar{Y} = \theta + b + \underbrace{\frac{e_1 + \dots + e_n}{n}}_{\bar{e}}$$

cancellation of ⊕, ⊖ errors . . .

③

... means that $\bar{e}$ is likely to be a lot closer to 0 than any of the individual errors $e_1, \dots, e_n$ going into $\bar{e}$

this is why averaging replicate observations is good

as n increases, $\bar{e}$ gets closer & closer to 0, meaning that $\bar{y}$ gets closer & closer to $(\theta + b)$

(truth) + (bias)

with an unbiased good measuring process, replication followed by averaging is guaranteed to get you closer to the truth

④

pop

conceptual: all possible measurements

sample

the observed measurements

imaginary dataset

all possible $\bar{Y}$'s

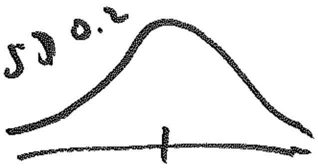
potassium

$N = \infty$

mean $\mu = 3.8$

SD $\sigma = 0.2$

SD 0.2



3.8

pop hist.

potassium

Y_1

$\vdots$

Y_n

$n = 4$

IID

mean $\bar{Y} = ?$

(ex. 3.45)

IID

$n=4$

mean $\bar{Y} = ?$

(ex. 3.9)

$\vdots$

3.45

3.9

$\vdots$

$M \to \infty$

long run mean

EV of $\bar{Y}$ $= \mu = 3.8$

long run SD

SE of $\bar{Y}$ $= 0.1$

SE of $\bar{Y}$ =

$$\frac{\sigma}{\sqrt{n}} = \frac{0.2}{\sqrt{4}} = 0.1$$

long run hist

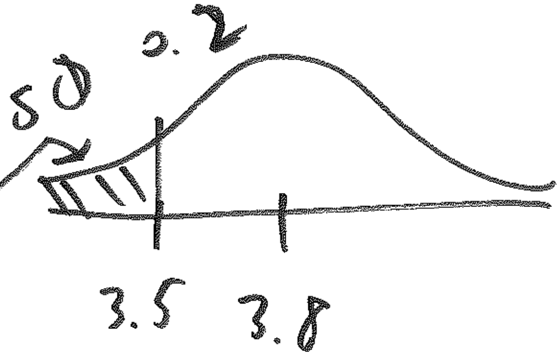
SE 0.1

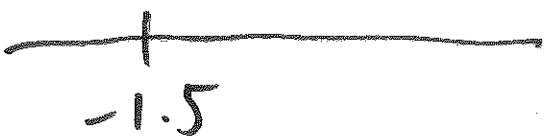
CLT

3.8

EV

(*)





$$\frac{3.5-3.8}{0.2} = -1.5$$

$.0668 \doteq 7\%$

hist of 1 measurement at a time (pop)

P(misdiagnosis with only $n=1$ measurement) $\doteq 7\%$

P(misdiagnosis with $n=4$ measurements) $= ?$

$= P(\bar{Y} < 3.5$ with $n = 4)$

long-run mean of $\bar{Y}$ = expected value of $\bar{Y}$ ~~math fact~~

$= \text{EV of } \bar{Y} = \boxed{E_{IID}(\bar{Y}) = \mu} = 3.8$

long-run SD of $\bar{y}$ =

standard error of $\bar{y}$ =

SE of $\bar{y}$ = $\boxed{SE_{IID}(\bar{y}) = \frac{\sigma}{\sqrt{n}}}$ math ~~fact~~

N X

μ X

$\sigma \uparrow$ $SE(\bar{y}) \uparrow$

$n \uparrow$ $SE(\bar{y}) \downarrow$

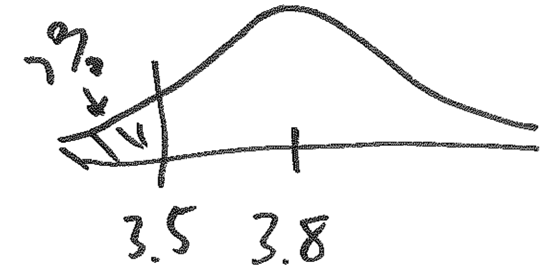
M X

$\bar{y}$ is a good (guess for) estimate of μ

SE of $\bar{y}$ = uncertainty of $\bar{y}$ as an estimate of μ

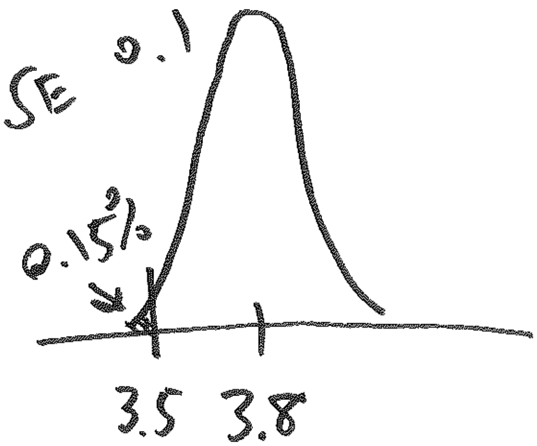
square root law: to cut your uncertainty about μ on the basis of $\bar{y}$ in half, you have to quadruple the sample size

SE 0.2



long run
hist of
$\bar{y}$ with $n=1$ (pop)

SE 0.1



long run
hist of
$\bar{y}$ with $n=4$

EV

$$\frac{3.5-3.8}{0.1} = -3$$

small prob = good

benefit

n	misdiagnosis probability	cost
1	7%	\$25
4	0.15%	\$100

pop

all individual crabs similar to those in this study

sample

the observed crabs

img. data

temperature

$N = ?$

(big)

like at random → like IID

temperature (°C)

Y_1 25.8

24.6

⋮

Y_n 25.4

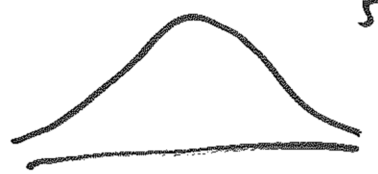
$n = 25$

mean $\bar{Y} = 25.0$

SD $s = 1.34$

mean $\mu = ?$

SD $\sigma = ?$



sample hist

?

pop hist.