this time: confidence intervals

next time: hypothesis testing

read: LN pp. L-161-173

today: LN pp. L-139 →

AMS 7
8 Jul 16

pop

known

the whole

the general

probability

at random →

easier

sample

unknown

the part

the particular

inductive reasoning

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pop

unknown

the whole

the general

statistical inference

harder

sample

known

the part

the particular

deductive reasoning

(2)

pop

all intertidal crabs similar to those in this study

sample

the observed crabs

imag data

all possible $\bar{y}$s

temperature

$N = ?$ (big)

mean $\mu = ?$

SD $\sigma = ?$

pop hist

actual

like IID

temperature (°C)

y_1 25.8

⋮

y_n 25.4

$n = 25$

mean $\bar{y} = 25.0$

SD $s = 1.34$

hypothetical

IID

sample hist

$n = 25$

mean $\bar{y} = ?$

(ex. 24.8)

25.0

24.8

⋮

$M \to \infty$

long run mean: EV of $\bar{y} = \mu$

est. long run SD: $\hat{SE}$ of $\bar{y} = \frac{s}{\sqrt{n}} \doteq 0.27$°C

long run hist: $\hat{SE}$ 0.27°C

μ

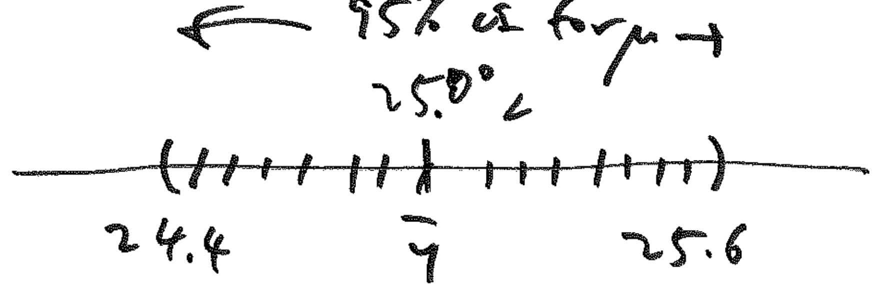
theory says $\mu = 24.3$°C $= \mu_0$

(10.04)

inferential summary

pop.	unknown pop. summary of main interest	μ = pop. mean equilibration temperature (°C)
sample data	estimate of μ	$\bar{y} = 25.0$°C
	give or take for $\bar{y}$ as estimate of μ	$\hat{SE}(\bar{y}) \doteq 0.27$°C
	95% confidence interval for μ	(24.4°C, 25.6°C)

I think that μ is around $\underline{\bar{y} = 25.0°C}$, give or take around $\underline{\hat{SE}(\bar{y}) = 0.27°C}$, and my 95% confidence interval for μ runs from $\underline{24.4°C}$ to $\underline{25.6°C}$



expected value of $\bar{y}$ = EV of $\bar{y}$ ④

$$= \boxed{E_{IID}(\bar{y}) = \mu}$$

math fact

---

standard error of $\bar{y}$ = SE of $\bar{y}$

$$= \boxed{SE_{IID}(\bar{y}) = \frac{\sigma}{\sqrt{n}}}$$

math fact

---

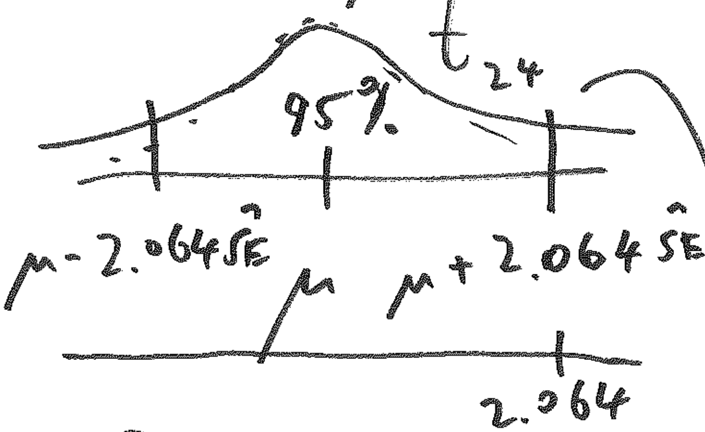
estimated SE of $\bar{y}$ $= \widehat{SE}_{IID}(\bar{y}) = \frac{s}{\sqrt{n}}$

"SE hat"

$$= \frac{1.34°C}{\sqrt{25}}$$

$$\doteq 0.27 °C$$

$\hat{SE}$ 0.27°C

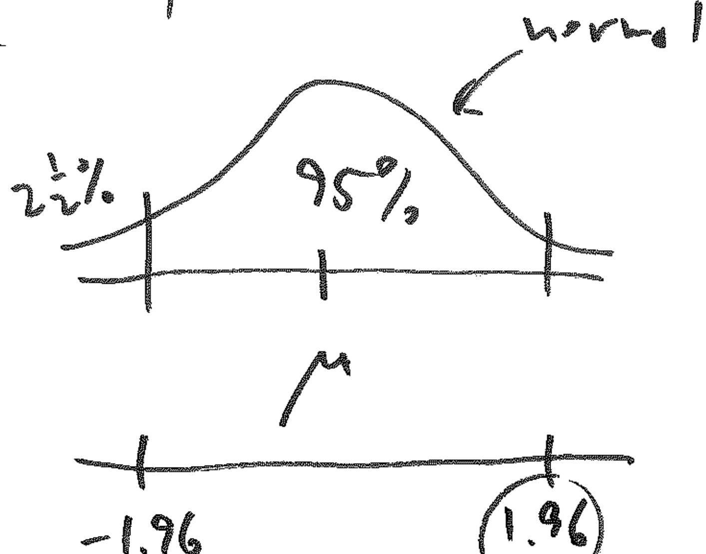


long-run hist of $\bar{Y}$, accounting for uncertainty about σ

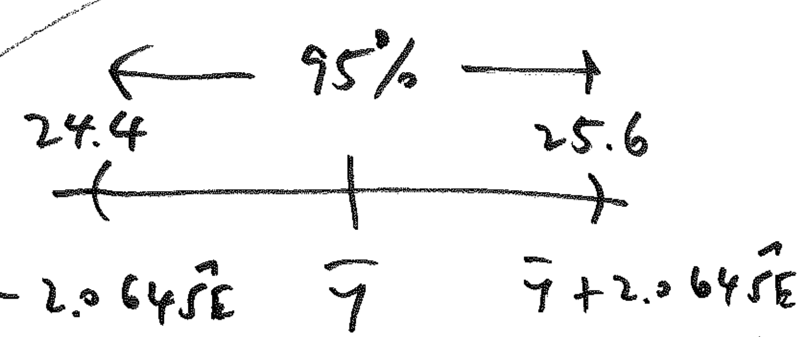
$t_{(n-1)}$ curve

degrees of freedom for using s as an estimate of σ

$(n=25)$



Neyman (1931)



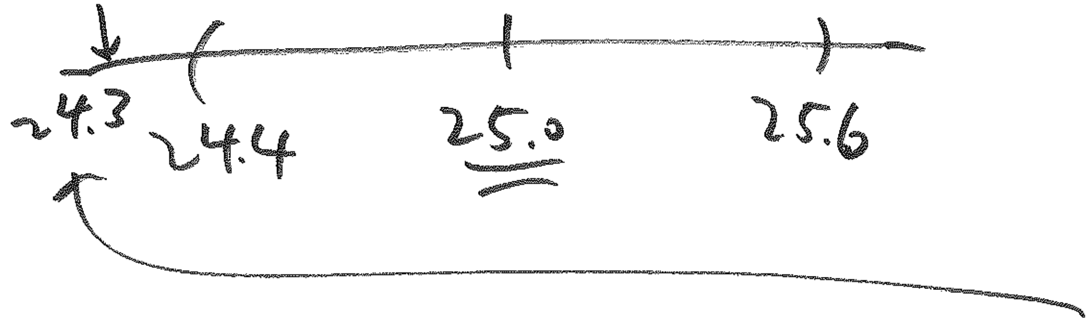
← 25.0°C

25.0 −.56   +.56   0.27°C

$\boxed{\bar{Y} \pm (t_{n-1,0.95})\frac{s}{\sqrt{n}}}$ is a 95% confidence interval for μ

(CI)

95% int. for μ



theory value for μ was 24.3°c

Since 24.3°c is not in the 95% interval for μ, we conclude that the data set does not support theory at 95% confidence level

When theory value μ_0 is outside the 95% interval, people say that the difference between $\bar{y}$ and μ_0 is <u>statistically significant</u> (statsig)

2 different notions of significance:

more important

$\boxed{q_1}$ (1) are 24.3°C & 25.0°C (practsig)

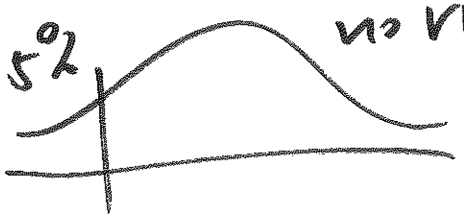
practically significantly different? $\boxed{A_1}$ think about whether the difference is big enough to matter in a practical, real-world sense

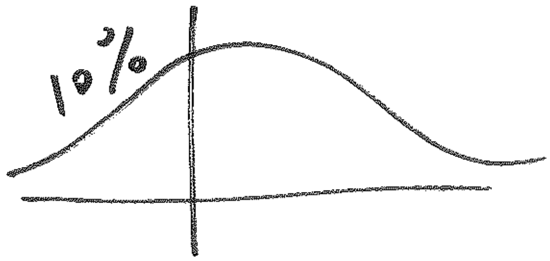
$\boxed{q_2}$ (2) are 24.3°C & 25.0°C

statsig different? $\boxed{A_2}$

see if μ_0 is in the 95% int for μ; if so, not statsig

not, statsig





SD of ~~pop~~ pop with only 2 values: 0 / 1

$= 1 \rightarrow \left(\text{larger value}\right) - \left(\text{smaller value}\right).$

$\sqrt{\left(\begin{matrix}\text{prop.}\\ \text{larger}\end{matrix}\right)\left(\begin{matrix}\text{prop.}\\ \text{smaller}\end{matrix}\right)}$

$p \qquad (1-p)$

$\boxed{\text{math fact.}}$

with 0/1 ~~pop~~ pop, $\sigma = \sqrt{p(1-p)}$

$\hat{p} = 83\%$

$\widehat{SE}(\hat{p}) = 11\%$

95% interval:

$\hat{p} \pm 1.96\ \widehat{SE}(\hat{p})$

$83\% \pm 2\ (11\%)$

our 95% CI is

(4.03) $(61\%, \cancel{105\%})$ 100%