AMS 7
Lecture 7
7/5/16

$\boxed{\text{Due Date}}$ for take-home midterm: Tue Secti

Roulette Problem

P(coming out ahead on any single play with strategy A) $= \frac{1}{38} = 2.6\%$

P(ahead with strategy B) $= \frac{2}{38} = \frac{1}{19} = 5\%$

Pop

all possibilities on any 1 spin

single # A ⑥

	net gain
1	-1
2	⋮
3	⋮
4	⋮
5	-1
6	+35
7	-1
⋮	⋮
36	-1
0	-1
00	-1

N = 38

mean: $\mu = -\frac{1}{19} = -0.05$

SD: $\sigma = 5.76$

Pop Hist:

at random w/ replacement (I.I.D.)

Sample

the observed spins

N = 1000

net gain
\$ -1
\$ +35
\$ -1
⋮
\$ -1

Sum \$ = ?
ex. $-\$61.00$

N = 1000

Sum \$ = ?
ex. $+\$11.00$

...

Imaginary Data S

possible sums (all)

-61
+11
⋮

$M \to \infty$

long-run

μ = expected value, $E_{1000}(S)$, EV of S

σ =

hist:

solutions p. 35 →

$\mu = \$52.60$

$\sigma = \$182$

SE: \$182

-53

hist 39% chance

33

μ: Every time the wheel spins with strategy A, I expect to win/lose around ____.
Lose $\mu = \$-0.05$

σ: Give or take around ____.
$\sigma = \$5.76$

Math Fact: any popl. having only 2 possible values, the pop SD is

$$\sigma = \left[\left(\text{larger value}\right) - \left(\text{smaller value}\right)\right]\sqrt{p\cdot(1-p)}$$

Here, smaller value $= -1$
larger value $= +35$
$p = \frac{1}{38}$ $\quad 1-p = 37/38$

$$\text{So } \sigma = \underbrace{[+35-(-1)]}_{+\$36}\sqrt{\frac{1}{38}\cdot\frac{37}{38}}$$

$$= \$5.76$$

Your net gain after $n=1000$ \$1 bets on a single # is like the SUM \$ of $n=1000$ IID draws from single-# population

real world
the model

To come out ahead, $\$ > 0$

To work out probability, need to sum (repetitons)

1. <u>Long Run Mean</u> = <u>Expected Value</u> of the sum S = EV of S = $\boxed{E_{IID}(S) = n\mu}$ math fac

in our example = (# draws)(pop mean)
= $1000 \cdot -.0626 = -\$52.60$

At the end of n=1, ... \$1 bets on single #
I expect to be ahead (behind) by
EV = - \$52.60, give or take ____.

2. <u>Long Run SD</u> of sum S in imag dataset = <u>Standard Error</u> of the sum S = SE of S

$\boxed{SE_{IID}(S) = \sigma \cdot \sqrt{n}}$

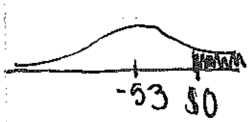
$\sigma \uparrow$ SE(S) $\uparrow$
n $\uparrow$ SE(S) $\uparrow$

in our example SE = $\$5.76 \cdot \sqrt{1000} = \$182$

3. <u>Long-Run Histogram</u>

SE = \$182

<u>Central Limit Theorem (CLT)</u>



$\frac{\$0 - (-53)}{\$182} \approx +0.29$ ~ 39% coming out ahead

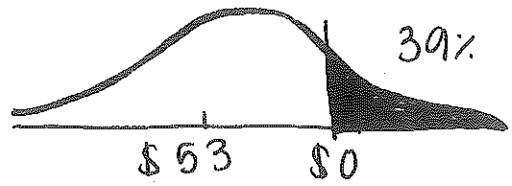
Standard Limit Theorem (CLT) (as $n \to \infty$)

Long run histogram of sum/mean of n number of IID draws from a population will look like normal curve.

The closer the pop histogram is to the normal curve is to begin with, the smaller that n must be to get a good normal approximation.

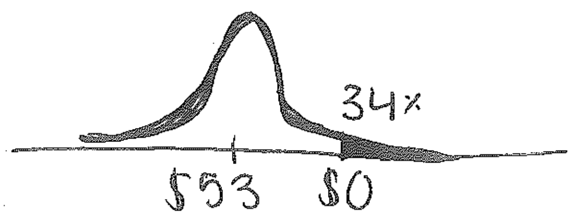
If pop histogram is normal to begin with, then the long-run histogram of S (or of mean) in imaginary data set is normal for all $n \geq 1$.

SE \$182



Long Run Hist.
of S (single #)

SE \$182



Long Run Hist.
of S (split)

risk seeking

$$\frac{\$(0-53)}{\$127} = +.42$$