Crab case study continued:

Let's compare theoretical mean 24.3°C and experimental sample 26.0°C.

Inductive Reasoning: whole -> part

"the part" = sample
"the whole" = pop

Deductive Reasoning: sample -> pop

Called "Statistical Inference"

Probability is easier than Statistical Inference.

Theory says \( M_o = 24.3°C \)

\[
\begin{array}{|c|c|}
\hline
\text{population} & \text{sample} \\
\hline
\text{unknown pop} & \text{estimate of } M \\
\text{summary of main interest} & (\text{around}) \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|}
\hline
\text{inference summary} & M = \text{pop mean equilibration } T \degree C \\
\hline
\text{confidence interval for } M & \bar{y} = 26.0°C \\
\text{give or take } \bar{y} \text{ as estimate of } M \\
\text{SE}(\bar{y}) = .27°C \\
\hline
\text{confidence interval for } M & 24.4°C \text{ to } 25.6°C \\
\hline
\end{array}
\]

I think that \( M \) is around \( \bar{y} = 26.0°C \), give or take around \( \text{SE}(\bar{y}) = .27°C \), and my confidence 24.4°C to 25.6°C.
Now let's build \( \text{imag data} \)

\[
\begin{bmatrix}
26.0 \\
24.8 \\
\vdots
\end{bmatrix}
\xrightarrow{M \to \infty}
\begin{bmatrix}
\tilde{Y} \\
\tilde{Y}_m
\end{bmatrix}
\]

Hypothetical

Long run mean (EV \( \tilde{Y} \))

Long run SD (\( \text{SE}(\tilde{Y}) \))

Long run hist

So let's fill in those blanks \( \uparrow \)

\[ E_{\text{iid}}(\tilde{Y}) = \mu \]

\[ \text{SE}(\bar{Y}) = \frac{\sigma}{\sqrt{n}} \]

but let's get a number...

Estimated SE of \( \bar{Y} \):

\[ \text{SE}_{\text{iid}}(\bar{Y}) = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{1.340 \degree C}{\sqrt{29}} \]

This is the "give or take" \( \rightarrow \)

\[ \text{SE} \approx 0.27 \degree C \]

\[ \bar{Y} \sim \text{normal} \]

Long run hist. \( \tilde{Y} \)

\[ T \text{ curve} \]

\[ n-1 \quad \text{degrees of freedom} \]

As \( s \) as estimate of \( \sigma \)

\[ -1.66 \quad 1.66 \]

\[ \bar{Y} - 2.064 \quad \bar{Y} + 2.064 \quad \text{SE}^2 \]

\[ 48 \]
\[ \bar{y} \pm (t_{n-1,\alpha/2}) \frac{s}{\sqrt{n}} \]

Neyman (1931)

Interval of confidence is a 95% confidence interval for \( \mu \).

24.4°C, 25.6°C

"The 95% interval"

The theory value for \( \mu \) was 24.3°C. Since 24.3°C is not in the 95% interval for \( \mu \), we conclude that the data set does not support theory at 95% confidence level.

When \( \mu_0 \) is outside 95% interval, people say the difference bw \( \bar{y} \) and \( \mu_0 \) is statistically significant (StatSig).

Significance:
1. Are 24.3°C + 260°C practically significantly different? If big enough to matter practically, more important, harder to judge.
2. Are 24.3°C + 260°C StatSig different? \( \mu_0 \) in 95% interval → not StatSig.

Highly StatSig → 99%.

CI are about the process of building intervals, not exact numbers.
Case Study: Maze w/ Rats

1 = Left  n=12 rats
0 = Right  mean 10/12 = 83%

Is it stat sig? Let's check (build interval)

(See L-157 for data sets)

note: \( \bar{x} = \hat{p} = \frac{10}{12} \)

SD of 0/1 (pop w/ only 2 values):
(larger value) - (smaller value) \( \sqrt{\frac{\text{prop}_{\text{large}} \cdot \text{prop}_{\text{small}}}{\hat{p} \cdot (1-\hat{p})}} \)

\[ \text{With pop 0/1 } \sigma = \sqrt{\text{prop}_{\text{large}} \cdot \text{prop}_{\text{small}}} \]

Then build inferential summary table (see L-158)

\[ \text{SE } \hat{p} \]

\[ \text{Hist?} \]

New variables in this problem...

\[ \text{SE}_{\text{prop}}(\hat{p}) = \sqrt{\frac{\text{prop}(1-\text{prop})}{n}} \]

Estimated SE use \( \hat{p} = \hat{p} \rightarrow \text{11%} \)

\( \hat{p} = 83\% \)  \( \text{SE}(\hat{p}) = 11\% \)  95% interval:

\[ \hat{p} \pm 1.96 \cdot \text{SE}(\hat{p}) \]

\[ 83 \pm 2(11\%) \]

\[ 81\% \text{ is } (81\%, 109\%) \rightarrow 100\% \]